Moore has proposed that the same moving wall criterion [Eq. (1)] applies to reverse flow separation, i.e., when the wall velocity moves counter to the edge velocity. The boundary-layer equation does not permit such a solution because a point where $\partial u/\partial y = u = 0$ and the condition that $u_w < 0$ and $u_e > 0$ also implies $\partial^2 u/\partial y^2 = 0$. As a result the inertia and viscous terms are zero at that point and the pressure gradient term would be unbalanced. Thus, there are no velocity profiles of the kind suggested by Eq. (1). Examination of the reverse flow similarity profiles[4] shows that there are no special characteristics which can be associated with separation such as an inflection point or an infinite displacement thickness and there is no tendency toward such conditions.

References

[1] Prandtl, L., "Über Flüssigkeitsberbegung bei sehr kleiner Reibung," *Proceedings of the Third International Mathematics Congress*, Heidelberg, 1904, pp. 484–491.

[2] Schlichting, H., *Boundary Layer Theory*, 6th ed., McGraw-Hill, New York, 1968, p. 203.

[3] Moore, F. K., "On the Separation of the Unsteady Laminar Boundary Layer," *Proceedings of Symposium on Boundary Layer Research, International Union of Theoretical and Applied Mechanics*, edited by H. Goertler, Springer-Verlag, Berlin, 1958, pp. 296–311.

[4] Danberg, J. E. and Fansler, K. S., "Similarity Solutions of the Boundary Layer Equations for Flow Over a Moving Wall," BRL Rept. 1714, April 1974, U.S. Army Ballistic Research Labs., Aberdeen Proving Ground, Md.

[5] Nachtsheim, P. R. and Swigert, P., "Satisfaction of Asymptotic Boundary Conditions in Numerical Solutions of Systems of Nonlinear Equations of Boundary Layer Type," TN D-3004, 1965, NASA.

Strong Interaction Associated with Transonic Flow Past Boattailed Afterbodies

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In order to estimate the performance of air breathing propulsive systems of a high speed aircraft operating under subsonic cruise conditions, it is necessary that the flowfield associated with transonic flow past boattails can be successfully studied and analyzed. This type of problem usually appears to be extremely difficult as the governing inviscid flow equation is of the mixed type. In addition, the relatively short boattail is usually immersed within the thick boundary layer of the approaching upstream flow so that the viscous interaction coupling the viscous and its external flows cannot be disregarded. Experimental investigation of such flows has been carried out, e.g., by Shrewbury.[1] His experimental data indicated considerable influence of the boattail juncture shape to the pressure distribution on the afterbody especially at high subsonic freestream Mach numbers which in turn illustrated the extremely sensitive character of the transonic flow.

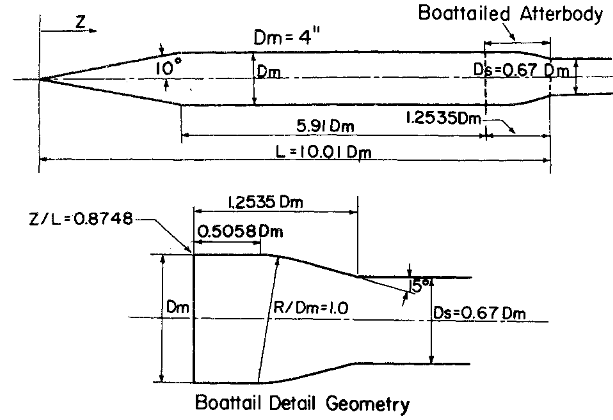

Fig. 1 Model geometry of the boattailed afterbody.

Since the suggestion of numerical treatment of inviscid transonic flow was made by Murman and Cole,[2] a considerable amount of activities in this area has been carried out. Krupp and Murman[3] computed transonic flow past lifting airfoils and slender bodies. Bailey[4] also extended the small disturbance treatment to calculate transonic flow past slender bodies of revolution. Steger and Lomax[5] and South and Jameson[6] employed the full potential equation to calculate the transonic flow past two-dimensional and axisymmetric bodies, respectively. It seems that the problem of transonic flow past boattails can be studied with these numerical relaxative schemes. Indeed, it is the intention here to report these results obtained from such a study. The preliminary calculations are restricted to a particular model configuration as shown in Fig. 1 which has been tested in the experimental program.[1] It was learned that the small disturbance treatment of the inviscid part of the transonic flow is not adequate even though the model appears to be relatively slender; thus, the full potential equation must be employed for its study. It will be seen that the "strong interaction" character of these problems within the transonic flow regime will be fully illustrated from the results obtained from this study even though the flow has not been separated away from the boattailed afterbody.

The detailed derivation and transformation of the basic equations for the inviscid, viscous flows and the description of numerical calculations will be presented in Ref. 7. Only a brief description of the method of calculations is presented here. For the invsicid flow, the basic equation for the disturbed velocity potential in the r, z coordinate system was transformed to[6]: $\xi = z$, $\zeta = B\eta/(1+B\eta)$ with $\eta = r - r_b(z)$ where B is a

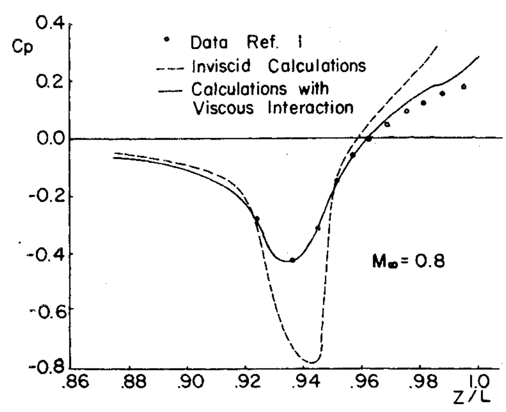

Fig. 2 Results of calculations and their comparison with the experimental data ($M_\infty = 0.8$).

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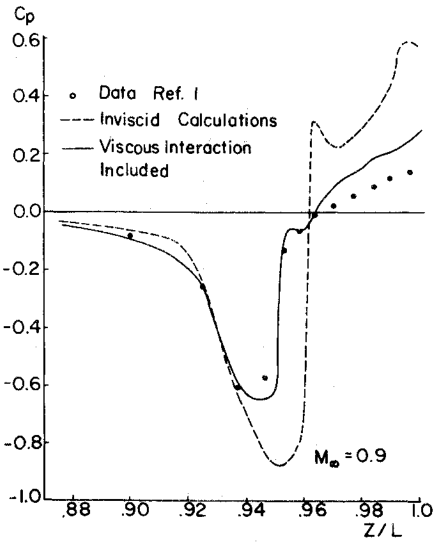

Fig. 3 Results of calculations for $M_\infty = 0.9$.

stretch parameter and r_b is the body profile in the meridian plane. With the proper boundary conditions, the disturbed potential function is calculated through the numerical relaxation scheme by performing finite difference calculations in the $\xi-\zeta$ plane. Different forms of the finite differences representing the partial differentiation in the ξ direction are employed depending upon whether the flow is locally supersonic or subsonic.[2] Final solution is reached when the successive change of the disturbed potential function is within a narrow limit ε (e.g., 3×10^{-6}) throughout the field of calculations. For the viscous flow, it was found that a simple description of the growth of the turbulent boundary- layer in the integral formulation by Sasman and Cresci[8] is adequate, and its effect on the inviscid flow can be accounted for through a displacement thickness correction of the body geometry so that the inviscid calculations are always based on the equivalent inviscid body configurations. Since the inviscid calculations are carried out by repeatedly scanning the field from upstream toward downstream flowfield, it is extremely expedient to perform the viscous calculations at the end of each sweep with the prevailing inviscid flow condition as its guiding freestream. Thus, a new equivalent body shape was established for the next inviscid calculations.

Figure 2 shows the results of such calculations for $M_\infty = 0.8$. The indicated inviscid results were of course obtained by bypassing the viscous flow calculations. It is obvious that the viscous flow effect was playing an equally important role in the solution of the problem and the final results agreed very well with the experimental data. The strong interaction character of the transonic flow past boattailed afterbody is thus fully illustrated.

Similar calculations have been carried out for cases of $M_\infty =$ 0.56 and 0.7 and the final results agreed again very well with the experimental data. For $M_\infty = 0.9$, it was found that "damping" to the change of the equivalent inviscid geometry was necessary before convergent calculations can be achieved. The results for this flow case are shown in Fig. 3. Agreement with the data is reasonably good, although slightly higher pressure level on the last portion of the boattail and a slight overshoot from the shock recompression have been observed.

References

[1] Shrewsbury, G. D., "Effect of Boattail Juncture Shape on Pressure Drag Coefficient of Isolated Afterbodies," TMX-1517, 1968, NASA.

[2] Murman, E. M. and Cole, J. D., "Calculation of Plane Steady Transonic Flows," *AIAA Journal*, Vol. 9, No. 1, Jan. 1971, pp. 114–212.

[3] Krupp, J. A. and Murman, E. M., "Computation of Transonic Flows past Lifting Airfoils and Slender Bodies," *AIAA Journal*, Vol. 10, No. 7, July 1972, pp. 880–886.

[4] Bailey, F. R., "Numerical Calculation of Transonic Flow about Slender Bodies of Revolution," TN D-6582, Dec. 1971, NASA.

[5] Steger, J. L. and Lomax, H., "Numerical Calculation of Transonic Flow about Two-Dimensional Airfoils by Relaxation Procedures," *AIAA Journal*, Vol. 10, No. 1, Jan. 1972, pp. 49–54.

[6] South, J. C., Jr. and Jameson, A., "Relaxation Solutions for Inviscid Axisymmetric Transonic Flow over Blunt or Pointed Bodies," *AIAA Computational Fluid Dynamics Conference Proceedings*, Palm Springs, Calif., July 19–20, 1973.

[7] Chow, W. L., Bober, L. J. and Anderson, B. H., "Numerical Calculations of Transonic Flow past Boattails," NASA Tech. Note, in preparation.

[8] Sasman, P. K. and Cresci, R. J., "Compressible Turbulent Boundary Layer with Pressure Gradient and Heat Transfer," *AIAA Journal*, Vol. 4, No. 1, Jan. 1966, pp. 19–25.

Asymptotic Post Buckling Solution of the Ring in an Elastic Foundation

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Introduction

In studying the post buckling of a simply supported strut on a linear Winkler elastic foundation, an interesting and apparently new result was reported in Ref. 1. Depending on certain geometrical parameters, the strut was found to possess either a stable or unstable symmetric point of bifurcation. More recently, it was shown that for the free ends and the semi-infinite simply supported strut only an unstable symmetric bifurcation exists,[2] and consequently imperfection sensitivity can be predicted.[3]

We are, therefore, inclined to ask ourselves whether a similar effect exists for a similar and (moreover) practically important problem like that of an elastically impeded circular ring under external pressure. In the following section we will try to answer this question.

Results of the Analysis

Considering a circular ring of the radius r, the axial stiffness EF, and the bending rigidity EI under a constant directional type of external pressure P, it is easy to show that the potential energy functional is given by

$$V = \int_0^{2\pi} \{\tfrac{1}{2}EI(\dot\psi^2/r) + \tfrac{1}{2}EF\varepsilon^2 r + Pr^2[\cos\psi(1+\varepsilon)-1]\}\,d\phi \qquad (1)$$

where ε is the axial strain, ψ is the angle of rotation, $(\dot{\ }) = d(\ )/d\phi$ and ϕ is the angular coordinate.

Eliminating the passive function ε, using the equilibrium equation

$$\varepsilon + Pr/EF\cos\psi = 0$$

we obtain

$$V = \int_0^{2\pi} \{\tfrac{1}{2}EI(\dot\psi^2/r) + \tfrac{1}{2}(Pr/EF)^2(\cos\psi)^2 r + Pr^2[\cos\psi(1-Pr/EF\cos\psi)]\}\,d\phi \qquad (2)$$

The main difference between Eqs. (1) and (2) is that Eq. (2) is a totally symmetric functional which essentially will facilitate the

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